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FREE CONVECTIVE OSCILLATORY FLOW OF A POLAR FLUID THROUGH A POROUS MEDIUM IN THE PRESENCE OF OSCILLATING SUCTION AND TEMPERATURE

P. M. Patil[a] **and P. S. Kulkarni**[b]

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The aim of this work is to analyze a two-dimensional oscillatory free convective flow of an incompressible polar fluid through a porous medium bounded by an infinite vertical porous plate with oscillating suction and temperature at the wall. The governing equations are based on the volume averaging technique. Analytical expressions for the velocity, angular velocity, and temperature fields are obtained by using the regular perturbation technique. The analysis reveals a multiple boundary layer structure near the wall for the fields mentioned.

Keywords: *Free convection, oscillatory flow, polar fluid, porous medium, oscillating wall temperature.*

Introduction. In recent years, considerable interest is being shown in the study of thermally driven flows in porous media owing to their applications in several engineering and technological fields. An extensive review of the works on this topic was given in [1]. Apart from the works mentioned in [1], we can also note [2–4].

The aim of the present study is to investigate the oscillatory free convective flow of a polar fluid through a porous medium, when the latter is bounded by a vertical surface with oscillating suction and temperature. The flow is due to the buoyancy forces generated by the temperature gradient. The porous region is filled with water containing soluble and insoluble chemical materials. Such a fluid is modeled as a polar fluid. A similar problem was solved in [1] for an oscillating heat flux at the wall. As evident from the above, the problem discussed below differs from that considered in [1] only by the boundary condition for the temperature. Thus we shall use equations and method of their solutions identical to those used in [1].

Mathematical Formulation. We consider a two-dimensional oscillatory free convective flow of a polar incompressible fluid through a porous medium bounded by an infinite vertical porous plate with oscillating suction and temperature on the wall. The geometry of the flow problem is given in Fig. 1. Far away from the wall, the main stream velocity $U'_\infty(t)$ oscillates over a constant nonzero mean value U'_∞. The x' axis is taken along the plate and the y' axis is normal to it. Let $(u', v', 0)$ and $(0, 0, \omega')$ denote the velocity and angular velocity fields. It is assumed that the Boussinesq approximation is applicable to the present problem. That is to say, the variable fluid properties are negligible except for the buoyancy term, which is directly responsible for the fluid motion. Due to the assumption of infinite plate, the flow variables, except for the pressure p, are functions of y' and t' only. The governing equations for the boundary layer flow, which are the same as those used in [1] (these are equations of continuity, linear momentum, angular momentum, and energy, respectively), are given in [5, 6]:

$$\frac{\partial v'}{\partial y'} = 0 , \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + (\nu + \nu_r)\frac{\partial^2 u'}{\partial y'^2} + 2\nu_r \frac{\partial \omega'}{\partial y'} - \frac{\nu + \nu_r}{K'} u' + g\beta_0 (T' - T'_\infty) , \tag{2}$$

$$\frac{\partial \omega'}{\partial t'} + v' \frac{\partial \omega'}{\partial y'} = \frac{\gamma}{I}\frac{\partial^2 \omega'}{\partial y'^2} , \tag{3}$$

[a]Department of Mathematics, J. S. S. College of Science, Vidyagiri, Dharwad-580 004, Karnataka, India; email: dr_pmpatil@yahoo.co.in; [b]Department of Aerospace Engineering, Indian Institute of Science, Bangalore-560 012, India. Published in Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 6, pp. 1118–1124, November–December, 2009. Original article submitted June 23, 2008. The article has been shortened with the authors' consent.

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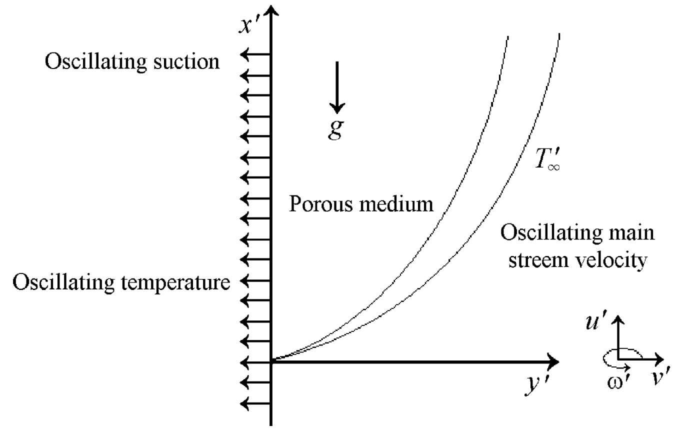

Fig. 1. Schematic diagram and coordinate system of the problem.

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}, \tag{4}$$

where $\gamma = (C_a + C_d)/I$.

Following D'ep [6], we write the relevant boundary conditions:

$$y' = 0: \quad u' = 0, \quad \frac{\partial \omega'}{\partial y'} = -\frac{\partial^2 u'}{\partial y'^2}, \quad T' = T'_w + \varepsilon \left(T'_w - T'_\infty\right) \exp(in't');$$

$$y' \to \infty: \quad u' \to U_\infty \left(1 + \varepsilon \exp(in't')\right), \quad \omega' \to 0, \quad T' \to T'_\infty. \tag{5}$$

For the oscillating suction at the wall, we assume

$$v' = -v_0 \left(1 + \varepsilon A \exp(in't')\right), \tag{6}$$

where v_0 is a nonzero constant mean suction velocity, ε is a small quantity, and A is a real positive constant such that $\varepsilon A \le 1$. Only the real parts of the physical quantities have physical meaning. The negative sign in Eq. (6) indicates that the suction velocity is directed towards the plate. The continuity equation (1) yields the solution

$$v'(y', t') = -v_0 \left(1 + \varepsilon A \exp(in't')\right), \tag{7}$$

which satisfies condition (6) at the wall.

In view of Eq. (7), Eqs. (2)–(4) become

$$\frac{\partial u'}{\partial t'} - v_0 \left(1 + \varepsilon A \exp(in't')\right) \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p}{\partial x'} + (\nu + \nu_r) \frac{\partial^2 u'}{\partial y'^2} + 2\nu_r \frac{\partial \omega'}{\partial y'} - \frac{\nu + \nu_r}{K'} u' + g\beta_0 (T' - T'_\infty), \tag{8}$$

$$\frac{\partial \omega'}{\partial t'} - v_0 \left(1 + \varepsilon A \exp(in't')\right) \frac{\partial \omega'}{\partial y'} = \frac{\gamma}{I} \frac{\partial^2 \omega'}{\partial y'^2}, \tag{9}$$

$$\frac{\partial T'}{\partial t'} - v_0 \left(1 + \varepsilon A \exp(in't')\right) \frac{\partial T'}{\partial y'} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}. \tag{10}$$

For the free stream, the momentum equation (2) takes the form

$$\frac{dU'_\infty}{dt'} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} - \frac{\nu + \nu_r}{K'}U'_\infty . \tag{11}$$

Eliminating the pressure gradient from Eqs. (8) and (11), we have

$$\frac{\partial u'}{\partial t'} - v_0\left(1 + \varepsilon A \exp(in't')\right)\frac{\partial u'}{\partial y'} = \frac{dU'_\infty}{dt'} + (\nu + \nu_r)\frac{\partial^2 u'}{\partial y'^2} + 2\nu_r\frac{\partial \omega'}{\partial y'} + \frac{\nu + \nu_r}{K'}(U'_\infty - u') + \beta_0 g(T' - T'_\infty) . \tag{12}$$

Introducing the following dimensionless quantities:

$$y = \frac{y'v_0}{\nu}, \quad t = \frac{v_0^2 t'}{\nu}, \quad n = \frac{\nu n'}{v_0^2}, \quad u = \frac{u'}{U_\infty}, \quad U = \frac{U'_\infty}{U_\infty}, \quad \omega = \frac{\nu\omega'}{U_\infty v_0}, \quad \alpha = \frac{\nu_r}{\nu}, \quad \beta = \frac{I\nu}{\gamma},$$

$$T = \frac{T' - T'_\infty}{(T'_w - T'_\infty)(1 + \exp(in't'))}, \quad \mathrm{Gr} = \frac{\nu g\beta_0(T'_w - T'_\infty)}{U_\infty v_0^2}, \quad K = \frac{K'v_0^2}{\nu^2}, \quad \mathrm{Pr} = \frac{\rho\nu C_p}{\lambda}, \tag{13}$$

we reduce Eqs. (12), (9), and (10) to equations in dimensionless form:

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A \exp(int)\right)\frac{\partial u}{\partial y} = \frac{dU}{dt} + (1 + \alpha)\frac{\partial^2 u}{\partial y^2} + 2\alpha\frac{\partial \omega}{\partial y} + \frac{1 + \alpha}{K}(U - u) + \mathrm{Gr}\,T , \tag{14}$$

$$\frac{\partial \omega}{\partial t} - \left(1 + \varepsilon A \exp(int)\right)\frac{\partial \omega}{\partial y} = \frac{1}{\beta}\frac{\partial^2 \omega}{\partial y^2} , \tag{15}$$

$$\frac{\partial T}{\partial t} - \left(1 + \varepsilon A \exp(int)\right)\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} . \tag{16}$$

The relevant boundary conditions (5) are reduced to

$$y = 0: \quad u = 0, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = 1 + \varepsilon \exp(int) ;$$

$$y \to \infty: \quad u \to U, \quad \omega \to 0, \quad T \to 0 . \tag{17}$$

For fluctuations in the main stream velocity U'_∞, we write

$$U'_\infty(t) = U_\infty\left(1 + \varepsilon \exp(in't')\right), \tag{18}$$

which after nondimensionalisation becomes

$$U = 1 + \varepsilon \exp(int) . \tag{19}$$

Equations (6) and (18) imply that the main stream velocity fluctuations are assumed to be in phase with the suction velocity fluctuations. As already noted, the present formulation of the problem differs from that in [1] only by the boundary condition for the temperature in (5) and by some quantities related to this condition.

Solution of the Problem. As in [1], the system of partial differential equations (14)–(16) is reduced to a system of ordinary differential equations by representing the fields of the linear and angular velocities and temperature as

$$u = u_0(y) + \varepsilon \exp(int)\, u_1(y) + o(\varepsilon^2) + \dots, \tag{20}$$

$$\omega = \omega_0(y) + \varepsilon \exp(int)\, \omega_1(y) + o(\varepsilon^2) + \dots, \tag{21}$$

$$T = T_0(y) + \varepsilon \exp(int)\, T_1(y) + o(\varepsilon^2) + \dots . \tag{22}$$

Substituting Eqs. (20)–(22) into Eqs. (14)–(16), respectively, equating the harmonic and nonharmonic terms, and disregarding coefficients of order $o(\varepsilon^2)$, we obtain the system of equations for the velocity, angular velocity, and temperature fields in the zeroth and first approximations (i.e., for steady-state and unsteady oscillatory flows). Here, we do not present these equations because they differ only slightly from those given in [1]. The same relates to the solutions of the equations mentioned, which are expressed by sums of exponential terms.

We denote u_1 and ω_1 as

$$u_1 = M_r + iM_i, \quad \omega_1 = W_r + iW_i$$

and introduce the quantities

$$|u_1| = \left(M_r^2 + M_i^2\right)^{1/2}, \quad |\omega_1| = \left(W_r^2 + W_i^2\right)^{1/2}$$

which are the magnitudes of velocity and angular velocity fluctuations, respectively. Then the quantities

$$\Phi_u = \arctan\frac{M_i}{M_r}, \quad \Phi_\omega = \arctan\frac{W_i}{W_r}$$

give the phase angles of velocity and angular velocity fluctuations with respect to suction and main stream velocity fluctuations. The mathematical expressions for M_r, M_i, W_r, and W_i are analogous to those given in Appendix of [1] and are not presented here.

The quantity of the uttermost physical interest is the skin friction at the wall due to its great importance in technological applications. On the basis of the velocity field in the boundary layer, the skin friction is obtained in the same manner as in [1].

Results and Discussion. The distributions of velocity u and angular velocity ω are calculated for various values of material parameters α and β, permeability K, Grashof number Gr, Prandtl number Pr, suction parameter A, and frequency n. The viscous (Newtonian) fluid case corresponds to $\alpha = 0$ and $\beta = 0$. The numerical results for the mean velocity u_0 and angular velocity ω_0 distributions are plotted in Figs. 2–6. In Fig. 2, the polar effect in the Darcy resistance is seen to increase the mean velocity and mean angular velocity up to 2.3% and 4.11%, respectively, in comparison to the results of [7] for a Newtonian fluid. From Fig. 3 we observe that the mean velocity and angular velocity (in magnitude) increase considerably near the wall with the material parameter α. The negative values of the angular velocity indicate that the microrotation of substructures in the polar fluid is clockwise.

The effect of the material parameter β on the velocities is seen from Fig. 4: as β increases, they decrease, which clearly indicates that the couple stresses are dominant during rotation of the particles. We should note that Figs. 2–4 bear close qualitative similarity to the corresponding figures from [1], differing only quantitatively. In Fig. 5, mean and angular velocity profiles are depicted for various values of Grashof number Gr. The free convection is set due to the temperature difference $T'_w - T'_\infty$. When $T'_w - T'_\infty < 0$ (i.e., Gr < 0)* and $T'_w - T'_\infty > 0$ (Gr > 0), the situations correspond respectively to heating and cooling of the porous plate by free convection currents. The numerical

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As in [1], we should note that dimensionless criteria, including the Grashof number, are usually positive quantities.

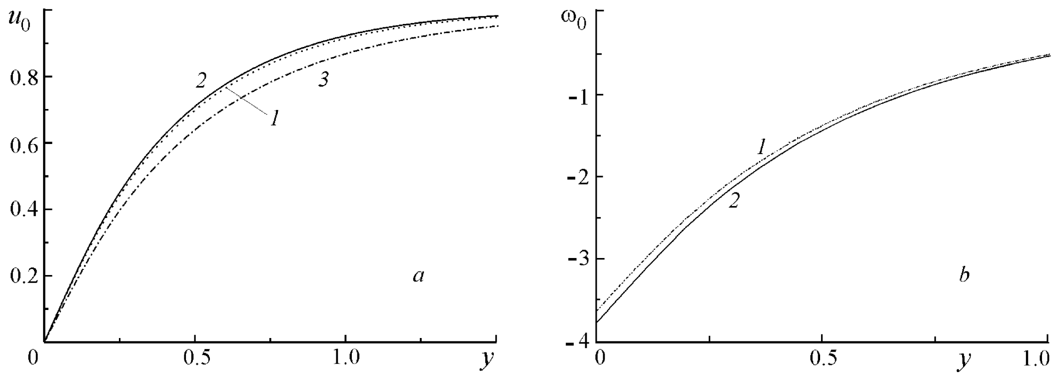

Fig. 2. Profiles of mean (a) and angular (b) velocities for the fluid in the case of consideration of the Darcy resistance without (1) and with (2) polar effects as well as for a Newtonian fluid (3).

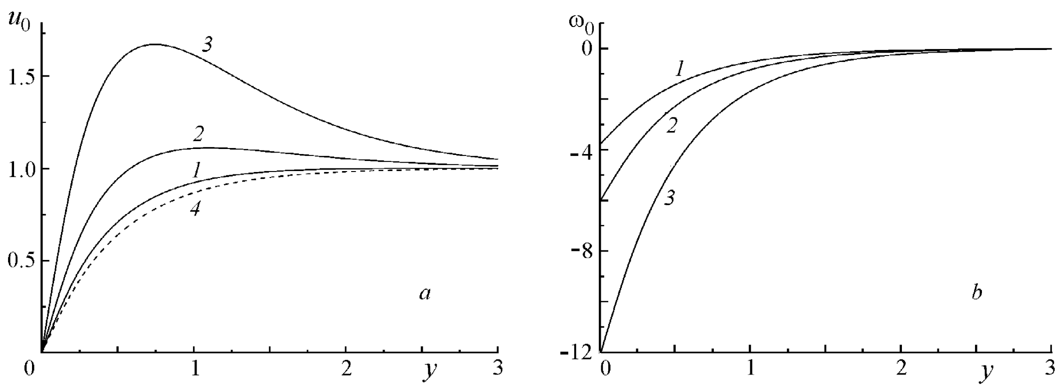

Fig. 3. Profiles of mean (a) and angular (b) velocities for different values of the material parameter α at $\beta = 2$, A = 0.2, Gr = 2, K = 0.5, and Pr = 7: 1) $\alpha = 0.1$; 2) 0.3; 3) 0.5; 4) 0.

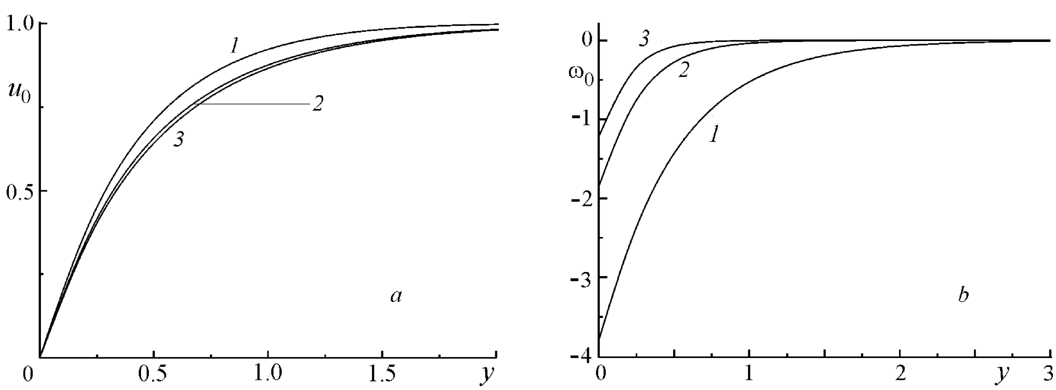

Fig. 4. Profiles of mean (a) and angular (b) velocities for different values of the material parameter β at $\alpha = 0.1$, $A = 0.2$, Gr = 2, $K = 0.5$, and Pr = 7: 1) $\beta = 2$; 2) 4; 3) 6.

results obtained for Gr = 0 correspond to the case of the absence of such currents. It is seen that, when heating of the surface is increased, the mean velocity falls, but the mean angular velocity changes sign (the latter situation differs from the results of [1]). For increasing cooling of the surface, the mean velocity as well as the mean angular velocity (in magnitude) increase.

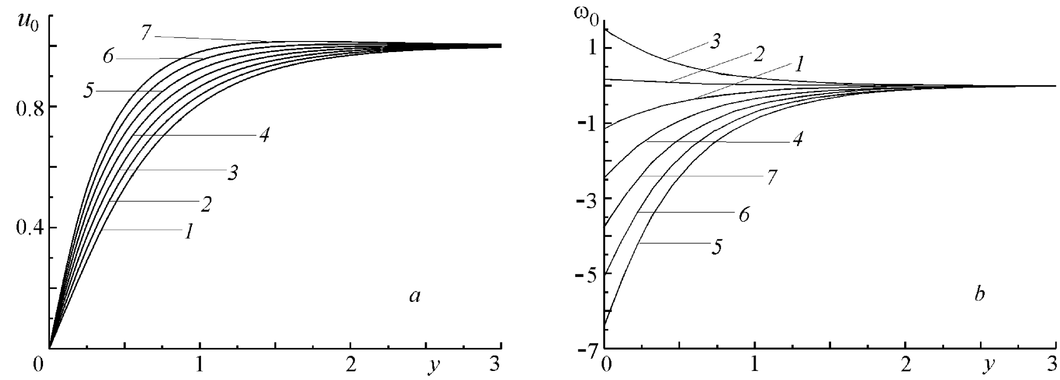

Fig. 5. Profiles of mean (a) and angular (b) velocities for different values of the Grashof number Gr at α = 0.1, β = 2, A = 0.2, K = 0.5, and Pr = 7: 1) Gr = –6; 2) –4; 3) –2; 4) 0; 5) 2; 6) 4; 7) 6.

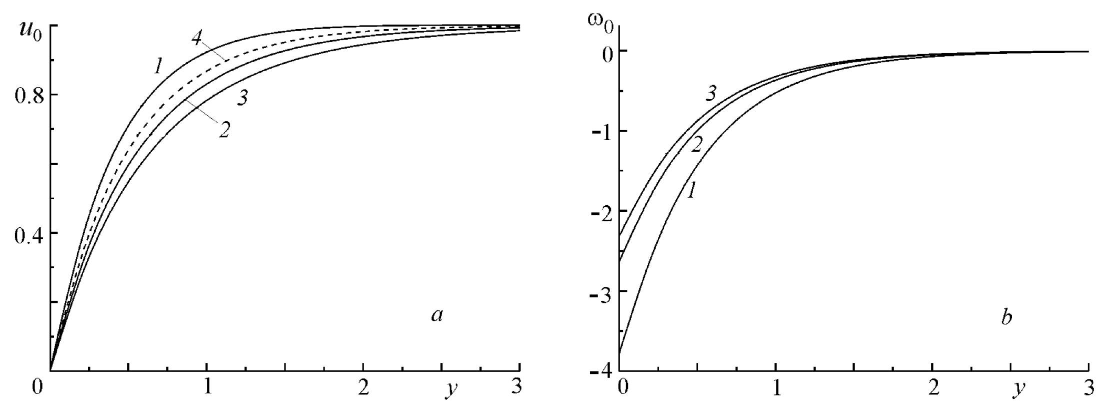

Fig. 6. Profiles of mean (a) and angular (b) velocities for different values of the permeability parameter K at α = 0.1, A = 0.2, β = 2, Gr = 2, and Pr = 7: 1) K = 0.5; 2) 1.5; 3) 3. The curve 4 corresponds to a Newtonian fluid.

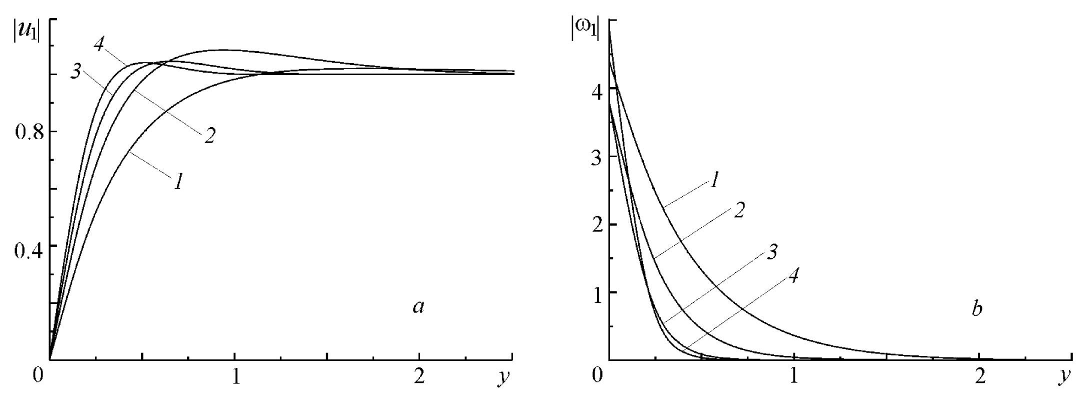

Fig. 7. Profiles of magnitudes of fluctuations of mean (a) and angular (b) velocities for different values of frequency at α = 0.1, A = 0.2, β = 2, Gr = 2, K = 0.5, and Pr = 7: 1) n = 1; 2) 10; 3) 50; 4)100.

In Fig. 6, the effect of the permeability K is given. It is seen that as K increases, the velocities decrease. The effect of the Prandtl number Pr is not presented here for the sake of brevity. However, it is seen from the numerical computation that an increase in the Prandtl number leads to a decrease in the velocities. Smaller values of Pr are equivalent to an increasing thermal conductivity, and hence heat is able to diffuse away from the heated plate more

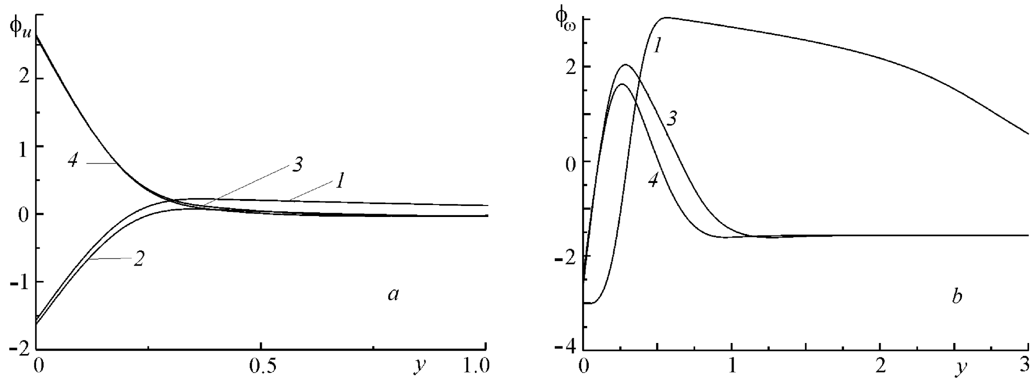

Fig. 8. Profiles of phase angle of fluctuations of mean (a) and angular (b) velocities for different values of frequency at α = 0.1, A = 0.2, β = 2, Gr = 2, K = 0.5, and Pr = 7: 1) n = 1; 2) 10; 3) 50; 4)100.

rapidly than in the case of higher values of Pr. Thus, the boundary layer is thicker and the rate of heat transfer is smaller for higher values of Pr. These results are the same as in [1].

The graphs for the magnitudes of velocity $|u_1|$ and angular velocity $|\omega_1|$ fluctuations and phase angles of these velocities ϕ_u and ϕ_ω are also not presented. As evidenced by calculations, the velocities increase with α, Gr, and A, but decrease with increasing β, K, and Pr. Further, it is noted that the suction parameter A exerts a profound influence on the flow. We have also considered the influence of all parameters involved on the existence of the phase lead or phase angle of both velocities. It is shown that there exist fluid layers where the angular velocity fluctuations are in phase with both the main stream and suction velocity fluctuations.

Figure 7 shows the profiles of $|u_1|$ and $|\omega_1|$ for different values of frequency n. It is seen that as n increases, $|u_1|$ increases, but for larger frequency the increase is more considerable near the wall surface and then $|u_1|$ decreases slowly. The same behavior is also observed in the case of $|\omega_1|$, i.e., the larger the value of n, the steeper the curve for $|\omega_1|$.

Figure 8 illustrates the behavior of ϕ_u and ϕ_ω for different values of frequency n. It is seen that ϕ_u has a phase lag ($\phi_u < 0$) near the wall for smaller frequencies, whereas a phase lead ($\phi_u > 0$) takes place for larger frequencies. As to ϕ_ω, this is characterized by a phase lag near the wall for both smaller and larger frequencies. The magnitude and phase angle of the skin friction are also obtained for different Gr. It is shown that in heating of the surface by free convection (Gr < 0), the skin friction increases more slowly as compared to the case of surface cooling.

The velocity, angular velocity, and temperature fields exhibit a multiple boundary layer structure. The thicknesses of these boundary layers are shown to be determined by the problem parameters.

Conclusions. We have examined the governing equations for an oscillatory free convective flow of a polar fluid through a porous medium in the presence of oscillating suction and temperature at the wall. It is shown that the flow characteristics are influenced by the material parameters α and β of the polar fluid, permeability K of the porous medium, Grashof number Gr, suction parameter A, and frequency n. It is worth noting that the results obtained are closely related to those for an oscillating heat flux at the wall. We suppose that the results presented, like the results of analogous work [1], could be useful in petroleum engineering.

NOTATION

A, suction velocity parameter; C_a, C_d coefficients of couple stress viscosities; C_p, specific heat at constant pressure; Gr, Grashof number; g, acceleration due to gravity; I, constant of dimensions of the moment of inertia of unit mass; K' and K, permeability and dimensionless permeability of the porous medium; M_r and M_i, real and imaginary parts of $|u_1|$; n' and n, frequency and dimensionless frequency of oscillations; Pr, Prandtl number; p, pressure; T' and T, temperature and dimensionless temperature in the boundary layer; T'_∞, temperature of the fluid far away from the plate; T'_w, temperature at the wall; T_1, fluctuating part of the temperature; t' and t, time and dimensionless time; U_∞, mean free stream velocity; U'_∞ and U, free stream velocity and dimensionless free stream velocities; u'

and v', velocity components along the plate and perpendicular to it; u, dimensionless velocity; u_0, mean velocity; u_1, fluctuating part of the velocity; v_0, mean suction velocity; W_r and W_i, real and imaginary parts of $|\omega_1|$; x' and y', coordinates along the plate and perpendicular to it; y, dimensionless coordinate normal to the plate; α, β, material parameters characterizing the polarity of the fluid; β_0, coefficient of volumetric expansion of the fluid; γ, spin gradient velocity; ε, perturbation parameter; λ, thermal conductivity of the fluid; ν, kinematic viscosity; ν_r, rotational kinematic viscosity; ρ, fluid density; ρ_∞, fluid density far away from the surface; ϕ_u and φ_ω, phase angles of fluctuations of the velocity and angular velocity; ω' and ω, angular and dimensionless angular velocity components; ω_0, mean angular velocity; ω_1, fluctuating part of the angular velocity. Subscripts: r and i, real and imaginary parts; w, wall; 0 and 1, steady basic and unsteady oscillatory flows.

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